# C_svdphoto

February 12, 2024

## 1 Worksheet C

***variationalform*** https://variationalform.github.io/

***Just Enough: progress at pace*** https://variationalform.github.io/

https://github.com/variationalform

Simon Shaw https://www.brunel.ac.uk/people/simon-shaw.

This work is licensed under CC BY-SA 4.0 (Attribution-ShareAlike 4.0 International)

Visit http://creativecommons.org/licenses/by-sa/4.0/ to see the terms.

This document uses python

and also makes use of LaTeX

in Markdown

### 1.1 What this is about:

This worksheet is based on the material in the notebooks

- matrices: matrix concepts and algebra
- systems: systems of linear equations, under- and over-determined cases
- decomp: eigensystem decomposition and SVD.

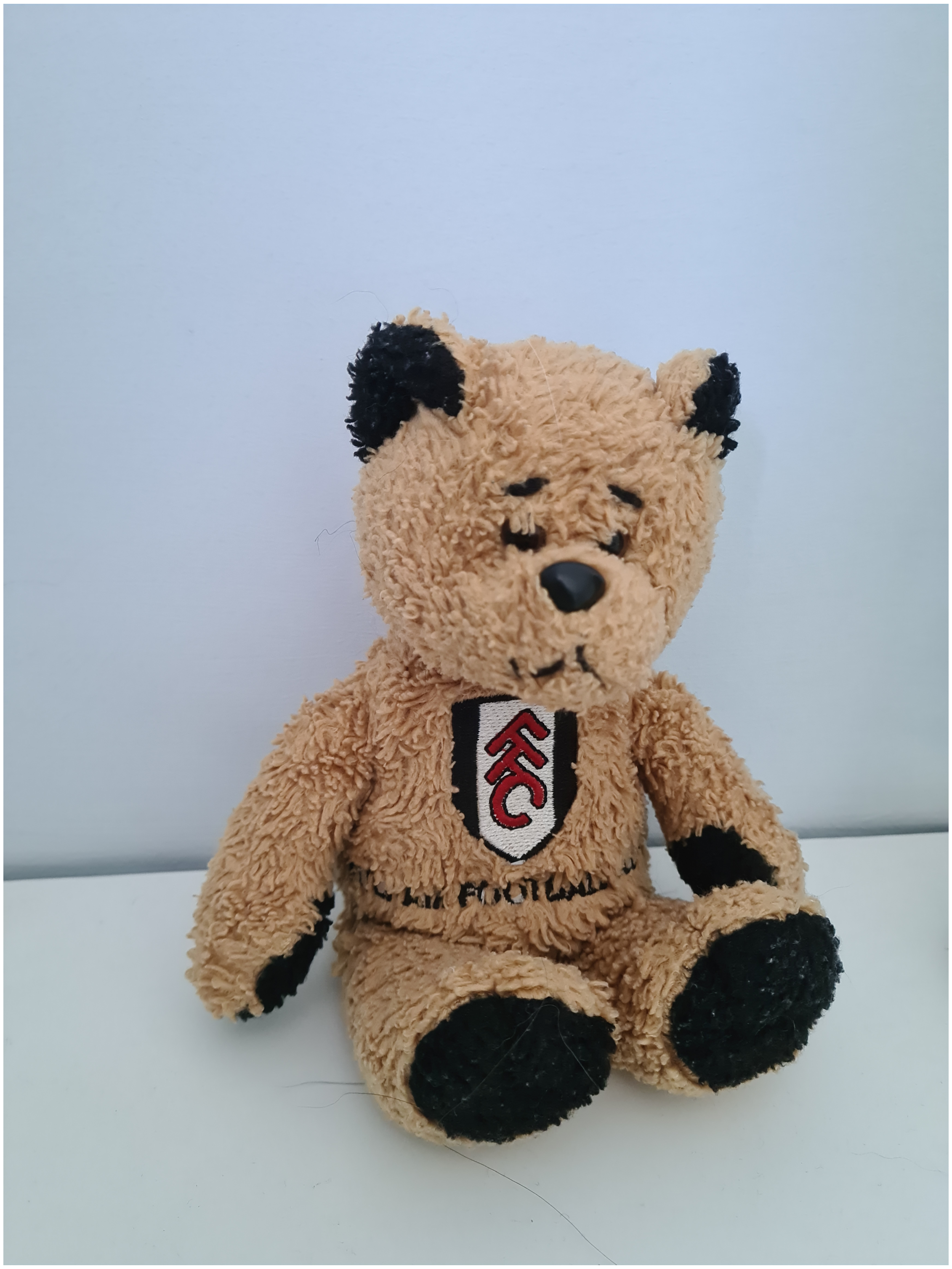
We look at a specific example of using SVD to compress a photograph.

Note that while the 'lecture' notebooks are prefixed with `1_`, `2_` and so on, to indicate the order in which they should be studied, the worksheets are prefixed with `A_`, `B_`, …

[1]:
```
# Two new imports here ... PIL and IPython
import matplotlib.pyplot as plt
import numpy as np
from PIL import Image
import IPython.display
```

[2]:
```
# Use a jpeg photo - ffc.jpg is about 6.2MB (use your own path/filename here)
IPython.display.Image(filename='./gfx/ffc.jpg', width = 150)
```

[2]:

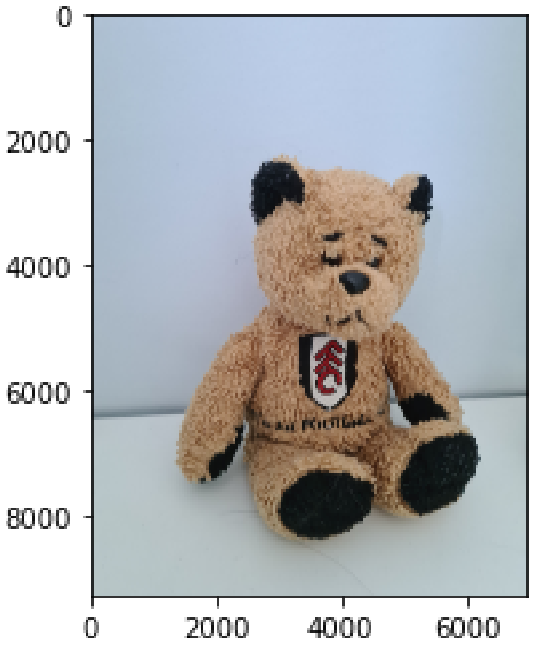


[3]:
```
# that is just a display, so ...
# load in the FFC bear - Roy - and visually check him.
```

```
img = Image.open('./gfx/ffc.jpg')
```

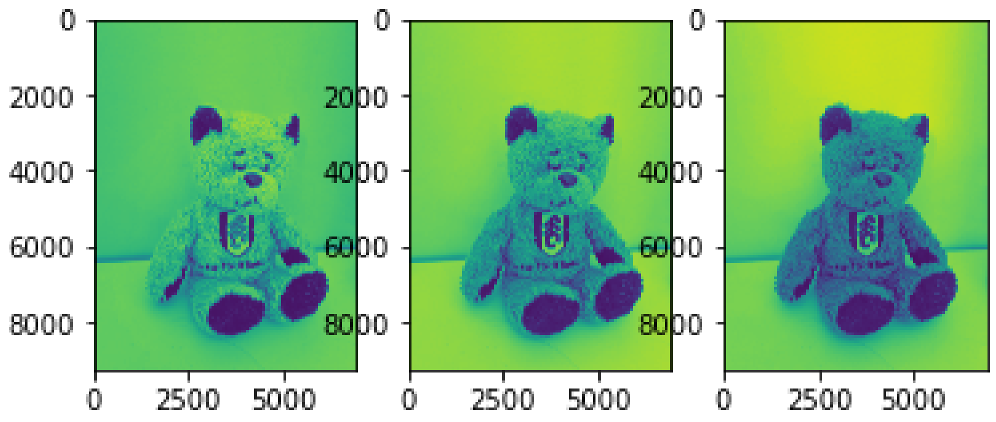
[4]:
```
# convert him to a numpy array for processing as a matrix
a = np.asarray(img)
im_orig = Image.fromarray(a)
plt.imshow(im_orig)
```

[4]:
```
<matplotlib.image.AxesImage at 0x7fc7d02f9668>
```



This image is made up of pixels where each pixel has a value for RED, GREEN and BLUE. We can get these *colour bands* and show them as follows...

[5]:
```
# convert band 'bnd' to a numpy array and show them...
for bnd in range(3):
  plt.subplot(1, 3, 1+bnd)
  img_mat = np.array(list(img.getdata(bnd)), float)
  img_mat = np.matrix(img_mat)
  img_mat.shape = (img.size[1], img.size[0])
  plt.imshow(img_mat)
```



[6]:
```
# get the red, green and blue bands as separate objects...
rband =img.getdata(band=0)
gband =img.getdata(band=1)
bband =img.getdata(band=2)

# and convert each to a numpy arrays for maths processing
imgr_mat = np.array(list(rband), float)
imgg_mat = np.array(list(gband), float)
imgb_mat = np.array(list(bband), float)
```

[7]:
```
# each of these is about 64k elements
print('sizes = ', imgr_mat.size, imgg_mat.size, imgb_mat.size)
print('shapes = ', imgr_mat.shape, imgg_mat.shape, imgb_mat.shape)
```

```
sizes =  64144128 64144128 64144128
shapes =  (64144128,) (64144128,) (64144128,)
```

[8]:
```
# get image shape - we can assume they are all the same
imgr_mat.shape = imgg_mat.shape = imgb_mat.shape = (img.size[1], img.size[0])
print('imgr_mat.shape = ', imgr_mat.shape)
print('imgg_mat.shape = ', imgg_mat.shape)
print('imgb_mat.shape = ', imgb_mat.shape)

# convert these 1D-arrays to matrices
imgr_mat1D = np.matrix(imgr_mat)
imgg_mat1D = np.matrix(imgg_mat)
imgb_mat1D = np.matrix(imgb_mat)
print(type(imgb_mat))
```

```
imgr_mat.shape =  (9248, 6936)
imgg_mat.shape =  (9248, 6936)
imgb_mat.shape =  (9248, 6936)
```

```
<class 'numpy.ndarray'>
```

- The chained assignment above is OK but you should be aware that there are pitfalls... Take a look here for example.
- https://stackoverflow.com/questions/7601823/how-do-chained-assignments-work
- It depends on what is being chained... Here they are just values and not objects...

[9]:
```
print(imgb_mat.shape is imgg_mat.shape)
```

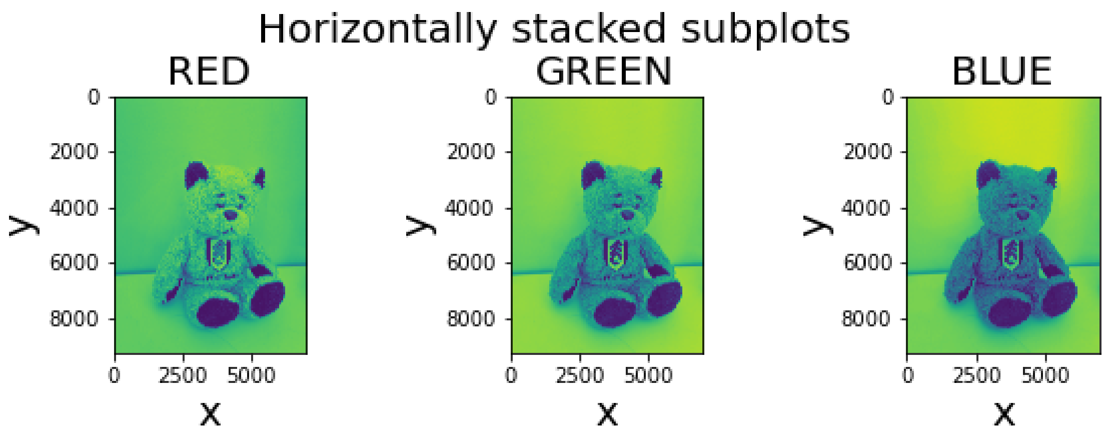
```
False
```

[10]:
```
# each of these is about 64 million elements: 9248 by 6936
print('sizes = ', imgr_mat1D.size, imgg_mat1D.size, imgb_mat1D.size)
print('shapes = ', imgr_mat1D.shape, imgg_mat1D.shape, imgb_mat1D.shape)
print('check: 9248 x 6936 = ', 9248 * 6936)
```

```
sizes =  64144128 64144128 64144128
shapes =  (9248, 6936) (9248, 6936) (9248, 6936)
check: 9248 x 6936 =  64144128
```

Let's look at these matrices - the following code introduces `subplot`

[11]:
```
fontsize=20
fig=plt.figure(figsize=(8, 3))
fig.suptitle('Horizontally stacked subplots', fontsize=fontsize)

axs=fig.add_subplot(1,3,1); axs.imshow(imgr_mat)
axs.set_xlabel('x', fontsize=fontsize); axs.set_ylabel('y', fontsize=fontsize)
axs.set_title('RED', fontsize=fontsize)

axs=fig.add_subplot(1,3,2); axs.imshow(imgg_mat)
axs.set_xlabel('x', fontsize=fontsize); axs.set_ylabel('y', fontsize=fontsize)
axs.set_title('GREEN', fontsize=fontsize)

axs=fig.add_subplot(1,3,3); axs.imshow(imgb_mat)
axs.set_xlabel('x', fontsize=fontsize); axs.set_ylabel('y', fontsize=fontsize)
axs.set_title('BLUE', fontsize=fontsize)
# use fractions of fontsize
plt.tight_layout(pad=0.3, w_pad=2.5, h_pad=0.3); plt.show()
```



Let's look at how much memory these photo layers occupy...

[12]:
```
print('Each matrix contains ', imgr_mat.size, ' elements')
print('Each element occupies ', imgr_mat.itemsize, ' bytes')
print('So each matrix occupies ', imgr_mat.size * imgg_mat.itemsize, ' bytes in␣
↪memory')
print('This is ', 3*imgr_mat.size * imgg_mat.itemsize, ' bytes for all three')

print('This is {:e} bytes for all three'.format(3*imgr_mat.size * imgg_mat.
↪itemsize))
```

```
Each matrix contains  64144128  elements
Each element occupies  8  bytes
So each matrix occupies  513153024  bytes in memory
This is  1539459072  bytes for all three
This is 1.539459e+09 bytes for all three
```

## 1.2 Compression...

Now that we know about the **Singular Value Decomposition**, we can hope to compress these objects using **mathematics**.

First get the SVD's of the R, G and B layers... (takes a while)

[13]:
```
Ur, Sr, VTr = np.linalg.svd(imgr_mat)
Ug, Sg, VTg = np.linalg.svd(imgg_mat)
Ub, Sb, VTb = np.linalg.svd(imgb_mat)
```

[14]:
```
print(f'RED:   shapes of Ur, Sr, VTr = {Ur.shape}, {Sr.shape}, {VTr.shape}')
print(f'GREEN: shapes of Ug, Sg, VTg = {Ug.shape}, {Sg.shape}, {VTg.shape}')
print(f'BLUE:  shapes of Ub, Sb, VTb = {Ub.shape}, {Sb.shape}, {VTb.shape}')
```

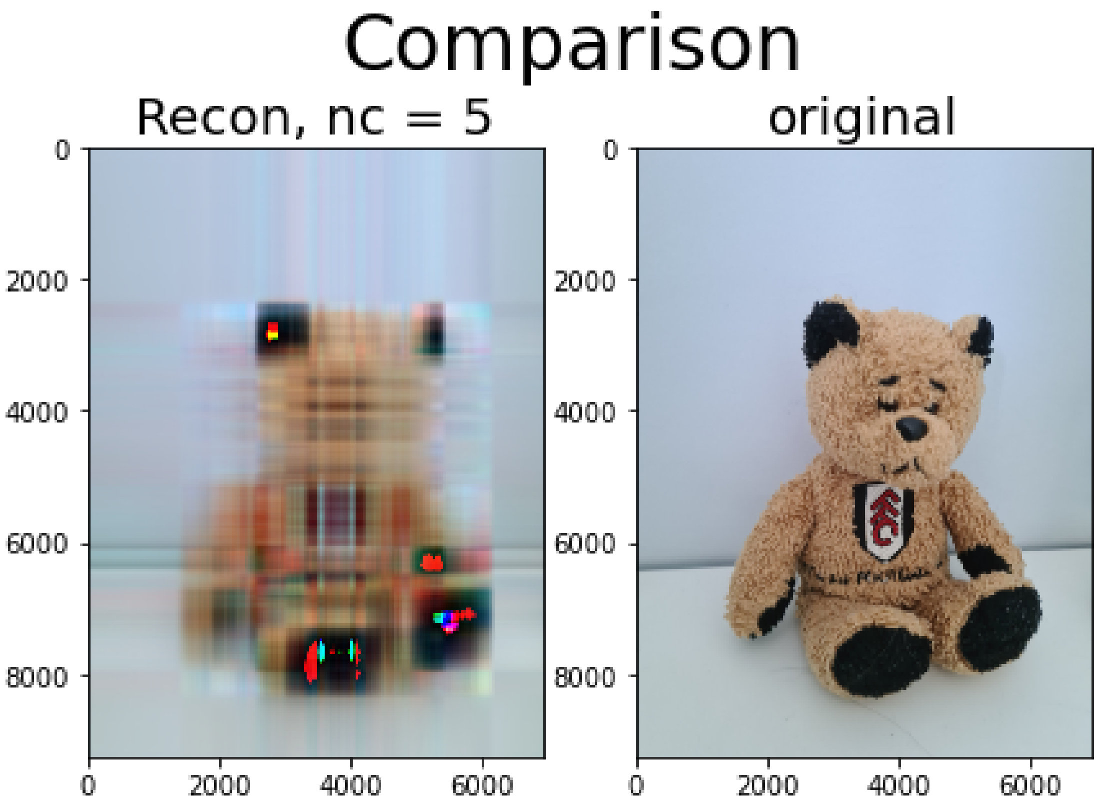
```
RED:   shapes of Ur, Sr, VTr = (9248, 9248), (6936,), (6936, 6936)
```

```
GREEN: shapes of Ug, Sg, VTg = (9248, 9248), (6936,), (6936, 6936)
BLUE:  shapes of Ub, Sb, VTb = (9248, 9248), (6936,), (6936, 6936)
```

[15]:
```
# choose the number of components to use in the reconstruction
nc = 5 # 1387
rec_imgr = np.matrix(Ur[:, :nc]) * np.diag(Sr[:nc]) * np.matrix(VTr[:nc, :])
rec_imgg = np.matrix(Ug[:, :nc]) * np.diag(Sg[:nc]) * np.matrix(VTg[:nc, :])
rec_imgb = np.matrix(Ub[:, :nc]) * np.diag(Sb[:nc]) * np.matrix(VTb[:nc, :])

img_all = np.array([rec_imgr, rec_imgg, rec_imgb]).T
img_all = np.swapaxes(img_all,0,1)

PIL_image = Image.fromarray(np.uint8(img_all)).convert('RGB')
# uncomment this to spawn an external viewer
#PIL_image.show()
# save the reconstruction
PIL_image.save("ffc_recon.jpg")
```

[16]:
```
fig=plt.figure(figsize=(7, 5))
fig.suptitle('Comparison', fontsize=30)

plt.subplot(1,2,1)
ax = plt.gca()
im = Image.fromarray(np.uint8(img_all)).convert('RGB')
ax.imshow(im)
ax.set_title(f'Recon, nc = {nc}', fontsize=20)

plt.subplot(1,2,2)
ax = plt.gca()
ax.imshow(im_orig)
ax.set_title('original', fontsize=20)
```

[16]:
```
Text(0.5, 1.0, 'original')
```



### 1.3 Review

We have seen a few examples now of how the SVD can play a very important role in Data Science. It is able to take data in matrix-form and distill it to its *essence*.

## 2 Exercises

1. Create logarithmic scree plots for each of the colour bands - what value of `nc` do these suggest?
2. Try to create non-log scree plots for each of the colour bands - what happens?
3. Calculate the percentage in memory saving resulting from the chosen value of `nc`
4. Look at each 'photo' corresponding to each singular component. Can you see the original coming through?
5. Overlay the first few modes for each colour. What can you see now?

[17]:
```
# Start solution 1 here
```

[18]:
```
# Start solution 3 here
```

[19]:
```
# Start solution 3 here
```

[20]:
```
# Start solution 4 here
```

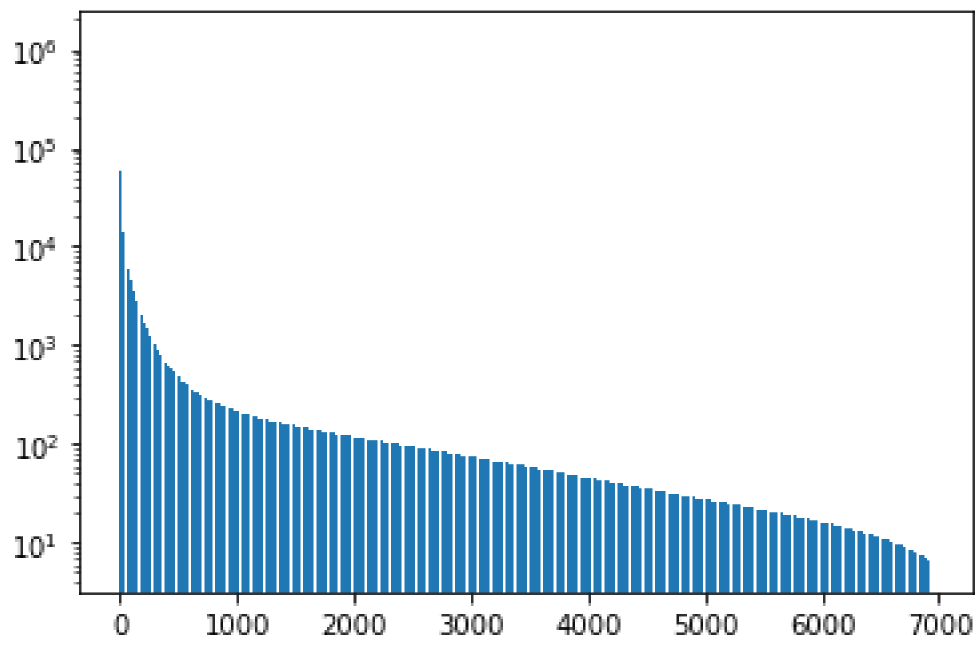
```
[21]: # Start solution 5 here
```

## 3 Outline Solutions

1. For the scree plots - here are the red ones.

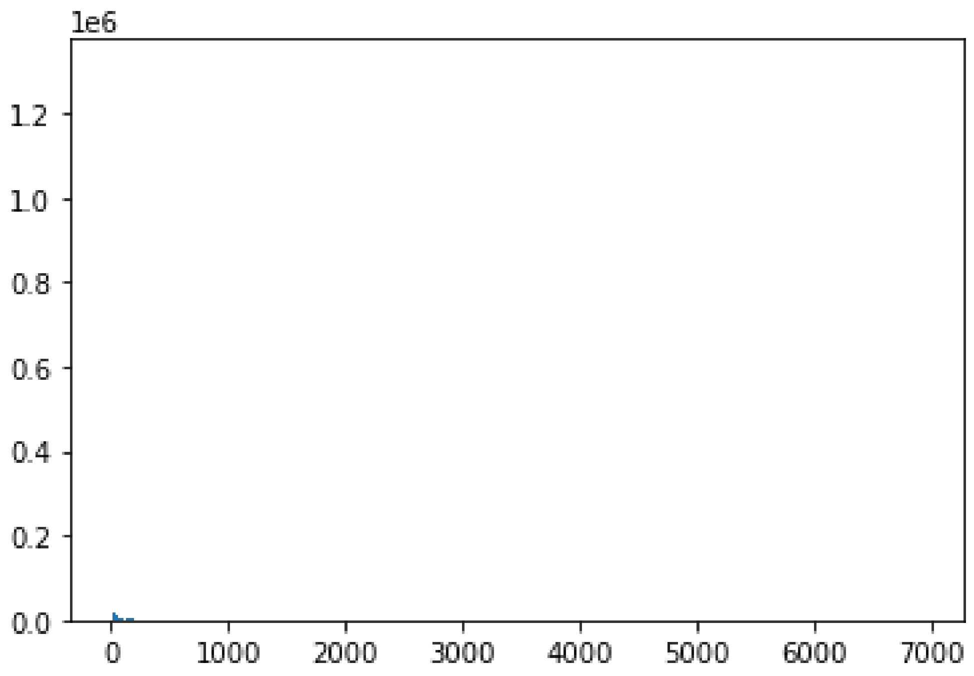
```
[22]: plt.bar(range(Sr.shape[0]),Sr, log=True)
```

```
[22]: <BarContainer object of 6936 artists>
```



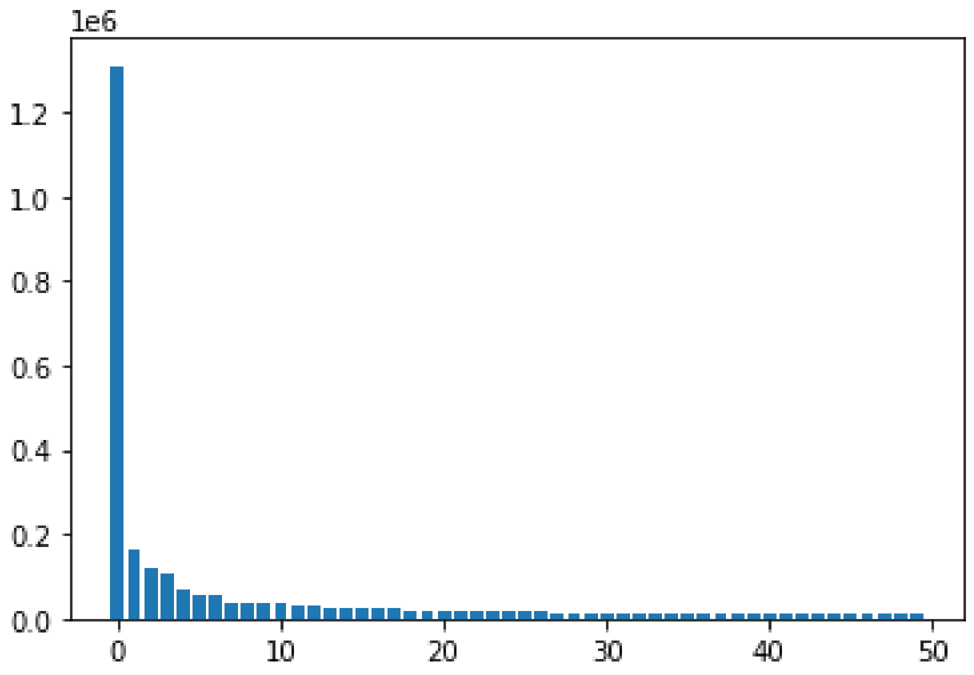
```
[23]: # this fails - there are too many and each bar is too thin to show
      #plt.figure(figsize=(12, 12)) # you can try a bigger plot, doesn't help
      plt.bar(range(Sr.shape[0]),Sr)
```

```
[23]: <BarContainer object of 6936 artists>
```



```
[24]: plt.bar(range(50),Sr[:50])
```

```
[24]: <BarContainer object of 50 artists>
```



2. The memory ratio is the new size of the first `nc` components divided by the size of the original

```
[25]: print('Each matrix contains ', imgr_mat.size, ' elements')
      print('Each element occupies ', imgr_mat.itemsize, ' bytes')
      print('So each matrix occupies ', imgr_mat.size * imgg_mat.itemsize, ' bytes in␣
       ↪memory')

      num_bytes_orig = 3*imgr_mat.size * imgg_mat.itemsize
      print(f'This is {num_bytes_orig} bytes for all three')
```

```
Each matrix contains  64144128  elements
Each element occupies  8  bytes
So each matrix occupies  513153024  bytes in memory
This is 1539459072 bytes for all three
```

For `nc` components, for each colour, we had this:

`rec_imgr = np.matrix(Ur[:, :nc]) * np.diag(Sr[:nc]) * np.matrix(VTr[:nc, :])`

So, for this colour we need `nc` columns in `Ur`, `nc` scalars in `Sr` and `nc` rows in `VTr`. And the same for each of the other colours.

```
[26]: print(f'RED:   shapes of Ur, Sr, VTr = {Ur.shape}, {Sr.shape}, {VTr.shape}')
      num_bytes_recon  = 3*nc*(Ur.shape[1] + 1 + VTr.shape[0])*Ur.itemsize
      print(f'This is {num_bytes_recon} bytes for all three')
```

```
RED:   shapes of Ur, Sr, VTr = (9248, 9248), (6936,), (6936, 6936)
This is 1942200 bytes for all three
```

```
[27]: # The percentage memory savings ratio is then...
      print(f'percentage savings in bytes = {100*num_bytes_recon/num_bytes_orig} %')
```

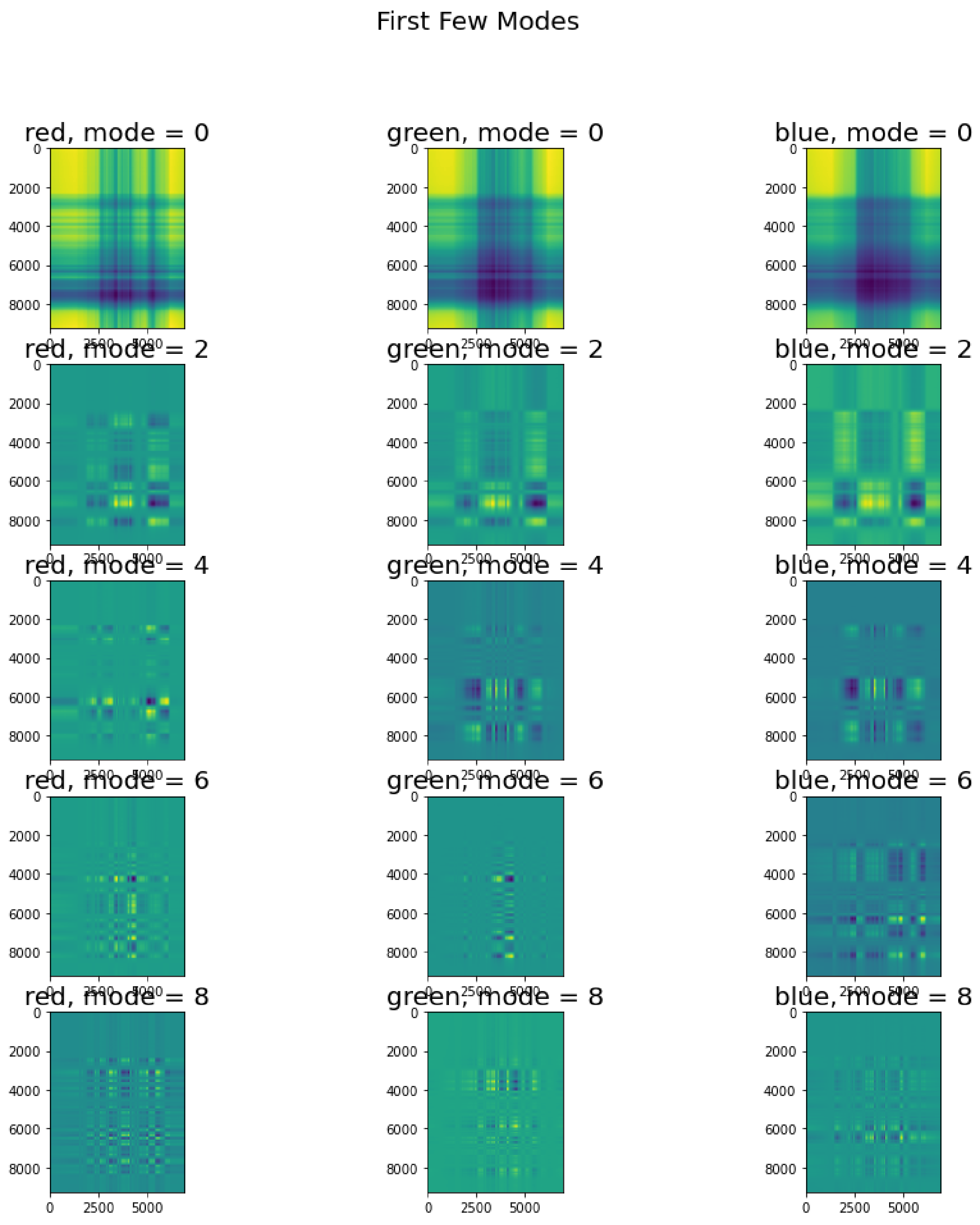
```
percentage savings in bytes = 0.12616119748326768 %
```

3. This is a complicated bit of code. There is a lot of value in understanding it

The graphics are unlikely to fit on the slide - use the notebook

```
[28]: # cell toggle scrolling from menu to prevent small window and scroll bar
      fontsize=20; fig=plt.figure(figsize=(15, 15))
      fig.suptitle('First Few Modes', fontsize=fontsize)

      c = 0; max_m = 5; step_m = 2
      for m in range(0,max_m*step_m, step_m):
        n = m+1
        c += 1
        mode_imgr = np.matrix(Ur[:, m:n]) * np.diag(Sr[m:n]) * np.matrix(VTr[m:n, :])
        axs=fig.add_subplot(max_m,3,c)
        axs.imshow(mode_imgr)
        axs.set_title('red, mode = '+str(m), fontsize=fontsize)
        c += 1
```

```
mode_imgg = np.matrix(Ug[:, m:n]) * np.diag(Sg[m:n]) * np.matrix(VTg[m:n, :])
axs=fig.add_subplot(max_m,3,c)
axs.imshow(mode_imgg)
axs.set_title('green, mode = '+str(m), fontsize=fontsize)
c += 1
mode_imgb = np.matrix(Ub[:, m:n]) * np.diag(Sb[m:n]) * np.matrix(VTb[m:n, :])
axs=fig.add_subplot(max_m,3,c)
axs.imshow(mode_imgb)

axs.set_title('blue, mode = '+str(m), fontsize=fontsize)
```



4. This code is also of value.

The graphics are unlikely to fit on the slide - use the notebook

```
[ ]: # cell toggle scrolling from menu to prevent small window and scroll bar
     fontsize=20
```

```
fig=plt.figure(figsize=(15,7))
fig.suptitle('First Few Modes Combined', fontsize=fontsize)

m = 0; n = max_m*step_m
c = 1
mode_imgr = np.matrix(Ur[:, m:n]) * np.diag(Sr[m:n]) * np.matrix(VTr[m:n, :])
axs=fig.add_subplot(1,3,c)
axs.imshow(mode_imgr)
axs.set_title('red, modes up to = '+str(n), fontsize=fontsize)
c += 1
mode_imgg = np.matrix(Ug[:, m:n]) * np.diag(Sg[m:n]) * np.matrix(VTg[m:n, :])
axs=fig.add_subplot(1,3,c)
axs.imshow(mode_imgg)
axs.set_title('green, modes up to = '+str(n), fontsize=fontsize)
c += 1
mode_imgb = np.matrix(Ub[:, m:n]) * np.diag(Sb[m:n]) * np.matrix(VTb[m:n, :])
axs=fig.add_subplot(1,3,c)
axs.imshow(mode_imgb)
axs.set_title('blue, modes up to = '+str(n), fontsize=fontsize)
```

```
[ ]: Text(0.5, 1.0, 'blue, modes up to = 10')
```

## 3.1 Technical Notes, Production and Archiving

Ignore the material below. What follows is not relevant to the material being taught.

### Production Workflow

- Finalise the notebook material above
- Clear and fresh run of entire notebook
- Create html slide show:
  - `jupyter nbconvert --to slides C_svdphoto.ipynb`
- Set `OUTPUTTING=1` below
- Comment out the display of web-sourced diagrams
- Clear and fresh run of entire notebook
- Comment back in the display of web-sourced diagrams
- Clear all cell output
- Set `OUTPUTTING=0` below
- Save
- git add, commit and push to FML
- copy PDF, HTML etc to web site
  - git add, commit and push
- rebuild binder

Some of this originated from

https://stackoverflow.com/questions/38540326/save-html-of-a-jupyter-notebook-from-within-the-r

These lines create a back up of the notebook. They can be ignored.

At some point this is better as a bash script outside of the notebook

```
[ ]: %%bash
NBROOTNAME='C_svdphoto'
OUTPUTTING=1

if [ $OUTPUTTING -eq 1 ]; then
  jupyter nbconvert --to html $NBROOTNAME.ipynb
  cp $NBROOTNAME.html ../backups/$(date +"%m_%d_%Y-%H%M%S")_$NBROOTNAME.html
  mv -f $NBROOTNAME.html ./formats/html/

  jupyter nbconvert --to pdf $NBROOTNAME.ipynb
  cp $NBROOTNAME.pdf ../backups/$(date +"%m_%d_%Y-%H%M%S")_$NBROOTNAME.pdf
  mv -f $NBROOTNAME.pdf ./formats/pdf/

  jupyter nbconvert --to script $NBROOTNAME.ipynb
  cp $NBROOTNAME.py ../backups/$(date +"%m_%d_%Y-%H%M%S")_$NBROOTNAME.py
  mv -f $NBROOTNAME.py ./formats/py/
else
  echo 'Not Generating html, pdf and py output versions'
fi
```